**Supplementary Material**

**Appendix 1: The full mathematical model of *Cydia pomonella***

1. ***Partial differential equation model of the stage-structured model***

Our mathematical model was motivated by previous works done on stage-structured models and renewal equations (Sharpe and Lotka 2009, Breda et al. 2012). For simplicity, we will present only the following stages: egg (, larvae (, the fifth instar larvae , the diapausing larvae , and adults (. We use partial differential equations as individuals can age out of a stage at any time.

Here, represent the density of individuals that are of age in their respective stage *j* at time . At each life-stage, the density of individuals changes due to the recruitment from the previous stage, maturation out to the next stage, and finally from stage-specific mortality and/or diapause processes. represents the per capita birth rate and represents the stage-specific per-capita mortality rate. is the thorough-stage development rate (senescence for the adults) which is determined by both age and time. Finally, fifth-instar individuals are induced into diapause at the rate and diapausing individuals are in the process of breaking dormancy at the rate . The vital rates are dependent on a temperature function that varies through time, .

The dynamics and the boundary conditions for each stage are shown below:

(1)

(2)

(3)

(4)

(5)

(6)

The total number of individuals within a stage can be calculated by taking the integral.

(7)

(8)

(9)

(10)

(11)

(12)

Differentiating Equations 7 - 12 then leads to:

(13)

(14)

(15)

(16)

(17)

(18)

Substituting in *Equations 1-6* into *Equations 13-18*.

(19)

(20)

(21)

(22)

(23)

(24)

Simplifying *Equations 19- 24* then leads us to:

(25)

(26)

(27)

(28)

(29)

(30)

***2. The development rate and linear chain trickery***

As the development rate is the crucial part of our phenology model, we shall focus on it intently here. The stage-specific development rate is described with which is the probability density describing the chance than an -aged individual in stage will mature out at time . is the cumulative probability of that describes the probability that an individual has not matured out of the stage by time . Therefore, the thorough stage development rate is given as:

(31)

To ensure that the individual has spent enough time in the life-stage by accumulating enough heat, the conditions are:

(32)

Here, represents the stage-specific instantaneous development rate and represents the time-varying stage duration.

Generally, mathematical models using ordinary differential equations assume that the dwelling time to be exponentially distributed such that individuals can mature out of their stage at any age. To incorporate developmental delays with individual variability, we use an Erlang distribution, a special case of the gamma distribution with an integer shape parameter and the rate parameter (Equation 32).

(33)

When , the Erlang distribution reduces to an exponential form and as goes to infinity, the Erlang distribution becomes a Dirac function which then transforms the integrodifferential equation into a series of discrete delay differential equations. The most biologically realistic form would be between the exponential and the Dirac function (Figure S1).

A picture containing screenshot

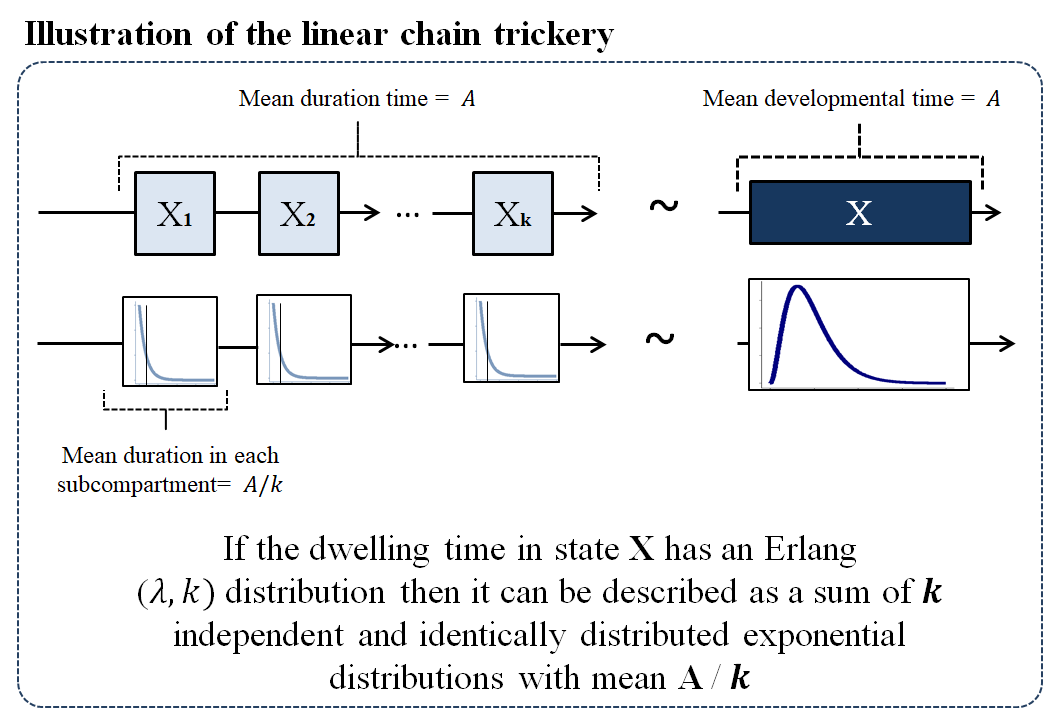
Description generated with high confidence

**Figure S1:** The probability distribution of the developmental times assuming an Erlang distribution with the mean age of developing out of a stage to be age 50. When = 1, the probability distribution becomes exponential (in purple). As approaches infinity, the Erlang distribution approaches the Dirac function (in black).

By substituting Equation 32 into Equation 31, the developmental rate then becomes.

(34) =

Unlike an exponential waiting time, the development rate is dependent on the age and must be jointly integrated with the density of individuals across all ages. However, equations 25-30 can be reduced to a system of ordinary differential equations through ‘linear-chain trickery’ (Figure S2) due to a special property of the Erlang distribution. Specifically, an Erlang distribution can be described as the summation of i.i.d exponential distributions. From the example below, we first assume that , the mean duration time in state to be Erlang distributed. Individuals must go through a series of subcompartments () with each subcompartment having an exponential distribution with the average sojourn time as . The individuals would flow through each subcompartments with the rate of .



**Figure S2:** An illustration describing the ‘linear chain trick’

*3. The full Cydia pomonella model with parameter estimates*

This is the full gamma-chain model used in the model. The full model has all five larval instars. Here represents the birth rate, represents the stage-specific instantaneous development rate (senescent rate for reproductive adults and diapause ‘development rate’ for diapausing larvae). is the stage-specific instantaneous mortality rate. represents the sub-stages for each of the life-stage.

(35)

(36)

(37)

(38)

(39)

(40)

(41)

(42)

(43)

(44)

**Table S1:** The full parameters for the *Cydia pomonella* model. Stars represent parameters estimated through maximum-likelihood.

|  |  |  |  |
| --- | --- | --- | --- |
| Function | Parameters | Value | Description |
| Birth Rate |  | 10.52 | Maximum birth rate at the optimum temperature |
|  | 24.62 | Optimum temperature (°C) for the maximum birth rate |
|  | 3.42 | Variability about the optimum temperature |
| C\* | 6.12 | Allele constant |
| Development Rate |  | 0.19 | Maximum development rate for Egg |
|  | 0.30 | Scalar determining the steepness for Egg |
|  | 18.59 | The temperature (°C) at which inflection occurs for Egg |
|  | 0.31 | Maximum development rate for Instar 1 |
|  | 0.33 | Scalar determining the steepness for Instar 1 |
|  | 17.60 | The temperature (°C) at which inflection occurs for Instar 1 |
|  | 0.45 | Maximum development rate for Instar 2 |
|  | 0.23 | Scalar determining the steepness for Instar 2 |
|  | 21.24 | The temperature (°C) at which inflection occurs for Instar 2 |
|  | 0.33 | Maximum development rate for Instar 3 |
|  | 0.35 | Scalar determining the steepness for Instar 3 |
|  | 18.46 | The temperature (°C) at which inflection occurs for Instar 3 |
|  | 0.30 | Maximum development rate for Instar 4 |
|  | 0.43 | Scalar determining the steepness for Instar 4 |
|  | 21.54 | The temperature (°C) at which inflection occurs for Instar 4 |
|  | 0.33 | Maximum development rate for Instar 5 |
|  | 0.21 | Scalar determining the steepness for Instar 5 |
|  | 20.95 | The temperature (°C) at which inflection occurs for Instar 5 |
|  | 0.09 | Maximum development rate for Pupae |
|  | 0.28 | Scalar determining the steepness for Pupae |
|  | 18.48 | The temperature (°C) at which inflection occurs for Pupae |
|  | 0.13 | Maximum senescent rate for R. Adults |
|  | 0.18 | Scalar determining the steepness for R. Adults |
|  | 20.22 | The temperature (°C) at which inflection occurs for R. Adults |
| Mortality Rate |  | 2.06 | Fitted scalar for Egg |
|  | 1.31 | Fitted scalar for Egg |
|  | 13.57 | The optimum temperature (°C) of survival for Egg |
|  | 5.00 | Fitted scalar for Larvae |
|  | 4.02 | Fitted scalar for Larvae |
|  | 16.24 | The optimum temperature (°C) of survival for Larvae |
| \* | 2.86 | Fitted scalar for Pupae |
| \* | 1.04 | Fitted scalar for Pupae |
| \* | 5.46 | The optimum temperature (°C) of survival for Pupae |
| \* | 2.86 | Fitted scalar for R. Adults |
| \* | 1.04 | Fitted scalar for R. Adults |
| \* | 20.00 | The optimum temperature (°C) of survival for R. Adults |
|  | \* | 6.49 | Fitted scalar for Diapausing Larvae |
|  | \* | 3.21 | Fitted scalar for Diapausing Larvae |
|  | \* | 3.72 | The optimum temperature (°C) of survival for Diapausing Larvae |
|  |  | 0.009 | The coefficient for density-dependence competition |
| Subcompartments |  | 50 | Number of sub-compartment for Egg |
|  | 30 | Number of sub-compartment for Instar 1 |
|  | 30 | Number of sub-compartment for Instar 2 |
|  | 30 | Number of sub-compartment for Instar 3 |
|  | 30 | Number of sub-compartment for Instar 4 |
|  | 30 | Number of sub-compartment for Instar 5 |
|  | 25 | Number of sub-compartment for Pupae |
|  | 25 | Number of sub-compartment for R. Adults |
|  | 25 | Number of sub-compartment for the Diapausing Larvae |
| Diapause  Induction | \* | 0.50 | Maximum diapause induction rate |
| \* | 0.50 | Scalar determining the steepness for Diapausing Larvae |
| \* | 220 | The temperature (°C) at which inflection occurs for Diapausing Larvae |
| Diapause Termination | \* |  | Maximum diapause termination rate |
| \* |  | Scalar determining the steepness for Diapausing Larvae |
| \* | 0.72 | The temperature (°C) at which inflection occurs for Diapausing Larvae |

**References:**

Breda, D., O. Diekmann, W. F. de Graafb, A. Pugliese, and R. Vermiglio. 2012. On the formulation of epidemic models (an appraisal of Kermack and McKendrick). Journal of Biological Dynamics 6:103–117.

Sharpe, F. R., and A. J. Lotka. 2009. L. A problem in age-distribution . The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science 21:435–438.